**2022 NDIA MICHIGAN CHAPTER
GROUND VEHICLE SYSTEMS ENGINEERING
AND TECHNOLOGY SYMPOSIUM
MODELING SIMULATION AND SOFTWARE (MS2) TECHNICAL SESSION
AUGUST 16-18, 2022 - NOVI, MICHIGAN**

A NEW APPROACH TO DYNAMIC FACTOR BASED ON VEHICLE ACTUAL VELOCITY AND TIRE SLIP

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ABSTRACT

The dynamic factor is a common characteristic of vehicle traction and acceleration performance derived from the traction balance of a vehicle. Typically, the dynamic factor is presented as a function of the vehicle theoretical velocity computed using the characteristics of the powertrain only with no tire slippage included. For off-road vehicles requiring large traction in most operational conditions, the tire slippage can impact considerably the vehicle velocity. Furthermore, tire slippages and vehicle actual velocity of multi-wheel drive vehicles significantly depends on the driveline system configuration. In this paper, a new method for analysis of the dynamic factor is proposed which includes the slippages of driving wheels and their influence on the vehicle actual velocity. The method facilitates determination of the effects of terrain grip limitations and slippage on the dynamic factor and acceleration performance of off-road vehicles. An example is given for a 4x4 vehicle, comparing the traditional and proposed approach.

Citation: J. Paldan, V. Vantsevich, D. Gorsich, J. Whitson, M. Letherwood, "A New Approach to Dynamic Factor Based on Vehicle Actual Velocity and Tire Slip," In *Proceedings of the Ground Vehicle Systems Engineering and Technology Symposium* (GVSETS), NDIA, Novi, MI, Aug. 16-18, 2022.

1. INTRODUCTION

During the conceptual design phase of both manned and autonomous military vehicles, design for terrain mobility should be further enhanced by correlating vehicle mobility indices, which characterize the tire-terrain interactions, to vehicle dynamic characteristics. In this regard, while maintaining adequate mobility, the vehicle should demonstrate sufficient traction and acceleration performance that is conventionally accepted as a vehicle operational property important for vehicle design.

A common, terramechanics- and vehicle dynamics-based characteristic of the traction and acceleration performance is the *dynamic factor* [1-6]. The mathematical expression for the dynamic factor is derived using the traction balance equation of vehicle moving in a straight-line motion. For a vehicle with n-number of drive axles and m-number of all axles ($m \geq n$), this equation presents the total circumferential wheel force of the vehicle, $F_{x\Sigma}$, which is the sum of the circumferential forces of the left and right driving wheels, $F'_{xi} + F''_{xi}$, that overcomes the rolling resistance, R'_{xi} and R''_{xi}, the vehicle weight component, $W_a sin\theta_n$, on a slope of θ_n, the vehicle inertia force presented by $\frac{W_a}{g}\delta_r a_a$ (here, δ_r is the rotating mass inertia factor, a_a is the vehicle acceleration), and, finally, the aerodynamic force, D_{ad}:

$$F_{x\Sigma} = \sum_{i=1}^{\mathrm{n}}(F'_{xi} + F''_{xi}) = \sum_{i=1}^{\mathrm{m}}(R'_{xi} + R''_{xi}) \pm W_a sin\theta_n + \frac{W_a}{g}\delta_r a_a + D_{ad} \quad (1)$$

Under an assigned velocity, the total circumferential force depends on the powertrain characteristics, and the aerodynamic force is determined by the vehicle design. The circumferential forces in Equation (1), $F'_{xi}, F''_{xi}, i = 1, n$, are computed using the powertrain characteristics only. Thus, the difference of $(F_{x\Sigma} - D_{ad})$ characterizes the total circumferential force margin/potential that can be created by the powertrain and may be utilized to overcome the rolling resistance, grade resistance, and develop a force needed to accelerate the vehicle. The difference $(F_{x\Sigma} - D_{ad})$ normalized by the gross weight of the vehicle, W_a, is known as the dynamic factor:

$$D_a = \frac{F_{x\Sigma} - D_{ad}}{W_a} = \frac{\sum_{i=1}^{\mathrm{n}}(F'_{xi} + F''_{xi}) - D_{ad}}{W_a} = \frac{\sum_{i=1}^{\mathrm{m}}(R'_{xi} + R''_{xi})}{W_a} \pm sin\theta_n + \frac{\delta_r}{g}a_a \quad (2)$$

From the general Equation (2), specific cases are obtained for steady state motion on a flat surface:

$$D_a = \frac{F_{x\Sigma} - D_{ad}}{W_a} = \frac{\sum_{i=1}^{\mathrm{n}}(F'_{xi} + F''_{xi}) - D_{ad}}{W_a} = \frac{\sum_{i=1}^{\mathrm{m}}(R'_{xi} + R''_{xi})}{W_a} \quad (3)$$

Steady state motion on a slope:

$$D_a = \frac{F_{x\Sigma} - D_{ad}}{W_a} = \frac{\sum_{i=1}^{\mathrm{n}}(F'_{xi} + F''_{xi}) - D_{ad}}{W_a} = \frac{\sum_{i=1}^{\mathrm{m}}(R'_{xi} + R''_{xi})}{W_a} \pm sin\theta_n \quad (4)$$

Acceleration on a flat surface:

$$D_a = \frac{F_{x\Sigma} - D_{ad}}{W_a} = \frac{\sum_{i=1}^{\mathrm{n}}(F'_{xi} + F''_{xi}) - D_{ad}}{W_a} = \frac{\sum_{i=1}^{\mathrm{m}}(R'_{xi} + R''_{xi})}{W_a} + \frac{\delta_r}{g}a_a \quad (5)$$

Equations (2) through (5) allow for characterizing the rolling resistance that can be overcome by a vehicle on the highest gear in transmission without switching to a lower gear and the maximal rolling resistance on the lowest gear. The equations facilitate determination of the range of velocities that correspond to stable work of the internal combustion engine at the full throttle on the highest gear. The dynamic factor is utilized to determine the vehicle maximal potential acceleration.

The dynamic factor also allows for ranging traction and acceleration capabilities of various vehicles by comparing the margins of the vehicle total circumferential forces, which are computed using the powertrain characteristics under the full throttle.

Graphically, in most vehicle dynamics studies, the dynamic factor is presented as a function of the vehicle theoretical velocity, V_a, which is computed using the characteristics of the powertrain only with no tire slippage included. For this reason, this dynamic factor is further referred in this report as “theoretical”, and the sub-index "a" is also utilized for the potential acceleration in Equations (1), (2) and (5). This approach works well for passenger car analysis on firm/asphalt roads when the use of lower gears in transmission is not significant. However, for off-road vehicles requiring large traction in most operational conditions, the tire slippage can impact considerably the vehicle velocity and, thus, the dynamic factor. For this reason, in references [7, 8], the vehicle actual velocity, V_x, is computed using the tire slippage.

Furthermore, the above-described approach for transiting from the vehicle theoretical to actual velocity in dynamic factor analysis cannot be utilized for vehicles with four or more driving wheels. Indeed, the wheels may have different tire slippages on different terrains and in various operational conditions. The tire slippages and vehicle actual velocity of multi-wheel drive vehicles significantly depends on the driveline system configuration, which plays a key role in the engine power distribution among the driving wheels. In conventional methods, the driveline characteristics are not included in dynamic factor analysis.

In this paper, a new method for analysis of the dynamic factor is proposed. The new method takes into account the effect of tire slippages of the driving wheels. The slippage influence on velocity is computed by using generalized parameters of the driveline system. The generalized parameters allow calculating a generalized slippage of the vehicle, $s_{\delta a}$, which relates the actual linear velocity of the vehicle V_x to the theoretical velocity V_a.

$$V_x = V_a(1 - s_{\delta a}) \tag{6}$$

Below, the proposed method is derived for the FED Alpha vehicle with a given configuration of the driveline system. The vehicle parameters are available at [9]. The method can be further extended in a similar way to any driveline system configuration of a vehicle with a given number of the wheels.

2. PROPOSED ACTUAL DYNAMIC FACTOR

The proposed dynamic factor is named "actual" because it is based on the actual velocity and acceleration of the vehicle, reduced by the tire slippage. The method for determining the slippage and its effect on the dynamic factor vs. velocity curves is given in this section. The analysis is performed for the driveline configuration shown in Figure 1, with open symmetric front and rear differentials and an open symmetric differential in the transfer case.

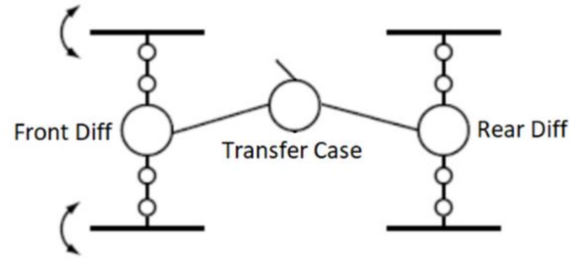

Figure 1: FED Alpha driveline configuration

An open differential always provides the same torques to its two output shafts (the friction in the differential is small and can be ignored). The wheel's rolling radius in the driven mode, r_w^0, is used for determining the circumferential force F_x from the torque T_w [10].

$$F_x = \frac{T_w}{r_w^0} \tag{7}$$

r_w^0 is the rolling radius of a wheel in the driven mode when the torque is zero; it can be determined experimentally or estimated from wheel load and tire characteristics and is considered known in this paper.

Thus, applying Equation (7) and the equality of the output torques for three open differentials in Figure 1 establishes the following relationships between the wheel circumferential forces:

$$\frac{F''_{xi} r^{0''}_{wi}}{F'_{xi} r^{0'}_{wi}} = 1,\ i = 1,2 \tag{8}$$

$$\frac{F''_{x2} r^{0''}_{w2} + F'_{x2} r^{0'}_{w2}}{F''_{x1} r^{0''}_{w1} + F'_{x1} r^{0'}_{w1}} = 1 \tag{9}$$

Each circumferential force relates to tire slippage through Equation (10) [11].

$$F'^{('')}_{xi} = \mu'^{('')}_{pxi} R'^{('')}_{zi} \left\{1 - \frac{s'^{('')}_{\delta ci}}{2 s'^{('')}_{\delta i}} \left[1 - \exp\left(-\frac{2 s'^{('')}_{\delta i}}{s'^{('')}_{\delta ci}}\right)\right]\right\} \tag{10}$$

here, $\mu'^{('')}_{pxi}$ is the peak friction coefficient, $R'^{('')}_{zi}$ is the normal reaction of a wheel, $s'^{('')}_{\delta i}$ is the tire slippage, and $s'^{('')}_{\delta ci}$ is the tire characteristic slippage. Parameters $\mu'^{('')}_{pxi}$ and $s'^{('')}_{\delta ci}$ are considered known and given as the terrain characteristics. The normal reactions are computed using a multi-body model of the vehicle. Thus, Equation (10) establishes a math function between the wheel circumferential force and tire slippage.

The theoretical velocity of the vehicle (before the velocity reduction from slippage) is calculated from the speed of the engine:

$$V_x = \omega_0 r_a^0 = \frac{\omega_e}{u_{tr}} r_a^0 \tag{11}$$

here, ω_0 is the angular velocity of the input shaft of the transfer case, ω_e is the angular velocity of the crankshaft, and u_{tr} is the gear ratio of the transmission that known for each gear. The generalized rolling radius of the vehicle in the driven mode, r_a^0, can be computed using Equations (12) and (13) derived for open differentials [10]:

$$r_{ai}^0 = \frac{2 r_{wi}^{0''} r_{wi}^{0'}}{(r_{wi}^{0''} + r_{wi}^{0'})},\ i = 1,2 \tag{12}$$

$$r_a^0 = \frac{r_{a1}^0 r_{a2}^0 (1 + u_d)}{r_{a2}^0 u_1 + r_{a1}^0 u_2 u_d} \tag{13}$$

r_{ai}^0 is the generalized rolling radius of an axle. u_d is the internal gear ratio of the differential (equal to 1 for the FED Alpha's symmetric differential).

The actual velocity is re-written as follows from Equation (6) and (11):

$$V_x = \omega_0 r_a^0 (1 - s_{\delta a}) = \frac{\omega_e}{u_{tr}} r_a^0 (1 - s_{\delta a}) \tag{14}$$

The generalized slippage, $s_{\delta a}$, is mathematically linked to the tire slippages through equations (15) and (16) derived for open differentials, where $s_{\delta ai}$ is the generalized slippage of an axle and $s_{\delta a}$ the generalized slippage of the vehicle [10]:

$$s_{\delta ai} = 1 - \frac{\left(r_{wi}^{0''} + r_{wi}^{0'}\right)(1 - s''_{\delta i})(1 - s'_{\delta i})}{r_{wi}^{0'}(1 - s'_{\delta i}) + r_{wi}^{0''}(1 - s''_{\delta i})},\ i = 1,2 \tag{15}$$

$$s_{\delta a} = 1 - \frac{(r_{a1}^0 u_2 u_d + r_{a2}^0 u_1)(1 - s_{\delta a1})(1 - s_{\delta a2})}{r_{a1}^0 u_2 u_d (1 - s_{\delta a1}) + r_{a2}^0 u_1 (1 - s_{\delta a2})} \tag{16}$$

The traction balance equation of vehicle moving in a straight-line motion is adapted from Equation (1):

$$\sum_{i=1}^{m} (R'_{xi} + R''_{xi}) + W_a \sin\theta_n + \frac{W_a}{g} \delta_r a_x + k_w A_f V_x^2 + F_d - \sum_{i=1}^{n} (F'_{xi} + F''_{xi}) = 0 \tag{17}$$

The actual acceleration a_x replaces the theoretical acceleration a_a. Aerodynamic force D_{ad} is replaced with its components: the frontal area projection of the vehicle A_f, the aerodynamic factor k_w, and the square of the actual velocity V_x. The aerodynamic factor is obtained from the following equation widely utilized in conventional vehicle dynamics:

$$k_w = 0.5 \rho C_x \tag{18}$$

ρ is the air density and C_x is the aerodynamic drag coefficient. Both A_f and k_w are considered given in this paper. Rolling resistances R_x in Equation (17) are dependent on the normal load on the wheels and on tire-terrain factors computed using a Bekker-Wong model of the terrain.

The weight W_a , gravity acceleration g, slope angle θ_n, and drawbar pull F_d are given and the rotating mass inertia factor δ_r can be computed based on recommendations in vehicle dynamics literature. Thus, the unknown variables in Equation (17) are the acceleration, velocity, and the circumferential forces of the wheels. The total circumferential force is split among all wheels from the torque provided by the engine:

$$\frac{\sum_{i=1}^{2} (F''_{xi} r_{wi}^{0''} + F'_{xi} r_{wi}^{0'})}{u_i u_{tc} u_{tr}} = T_e \tag{19}$$

here, u_{tc} is the gear ratio of the transfer case, u_i is the gear ratio of the driveline system from a wheel to the transfer case, which includes the gear ratio of the final drive. The engine torque, T_e, is related to the angular velocity of ω_e through a lookup table at wide-open throttle that is considered given, i.e., $T_e = f(\omega_e)$.

Finally, six equations, including Equations (17), (8), (9), (14), and (19) contain six unknowns, which are

- Acceleration, a_x
- Velocity, V_x
- Four tire slippages, $s_{\delta i}^{\prime(\prime\prime)}, i = 1,2$

while the generalized slippage, $s_{\delta a}$, and the forces, $F_{xi}^{\prime(\prime\prime)}$, are expressed through the tire slippages and the engine torque. The latter is given by the lookup table as a function of the engine angular velocity, which, in its turn, is included in Equation (14) for the vehicle actual velocity, V_x.

By solving the system of six Equations (8), (9), (14), (17), and (19), the dynamic factor from Equations (2) through (5) can be further established as a function of the vehicle actual velocity.

The dynamic factor of FED Alpha was computed moving with constant velocities on a flat meadow terrain that has a loam soil sublayer with moisture content varying from 8% to 12% and grass cover of 0.05m to 0.10m (case characterized by Equation (3)). The surface of motion is completely flat, i.e., there are no limitations imposed on the velocity due to actual surface bumps. The graphs in Figure 2 are for the conventional approach, in which the D_a-numerical values of the theoretical dynamic factor were computed using the powertrain characteristics on six gears in the transmission.

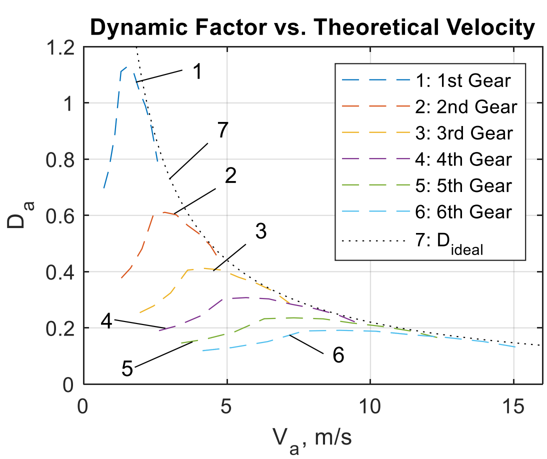

Figure 2: Dynamic factor of FED Alpha, Potential values not limited by grip, $\boldsymbol{D_a}$

The six curves are enveloped by the ideal dynamic factor given by hyperbolic curve 7:

$$D_{ideal} = \frac{N_{e\,max}\eta_m}{W_a V_a} \tag{20}$$

here, $N_{e\,max}$ is the engine max power, η_m is the mechanical efficiency of the transmission and driveline (assumed constant). The D_{ideal}-curve signifies the best dynamic performance that the vehicle could demonstrate due to the complete conversion of the engine power into the traction and velocity if the vehicle would have a continuously variable transmission. Indeed, all the D_{ideal} and V_a paired coordinates of the points on curve 7 being substituted in Equation (20) will result in the engine maximal power.

The dynamic factor of vehicles with mechanical gear transmissions takes lower magnitudes than given by the D_{ideal}-curve meaning that the engine power is underutilized. By increasing the number of gears in transmission, it becomes possible to move the theoretical dynamic factor on each gear nearly to the ideal curve and, thus, increase the utilization of power as shown in Figure 2. On each higher gear, if the resistance to motion increases and velocity drops, the gear should be switched to a lower gear at the maximal value of the dynamic factor. This allows for further increasing the dynamic factor of the vehicle and continuing its stable movement.

In Figure 2, the conventional theoretical dynamic factor is presented as it is computed using the powertrain characteristics. However, not all the D_a-values can be implemented to support vehicle traction and acceleration performance. Indeed, the maximum traction of the vehicle is limited by the tire-terrain grip, which limits wheel circumferential forces to a maximum dependent on $R_z^{\prime(\prime\prime)}$, $\mu_{px}^{\prime(\prime\prime)}$, and $s_{\delta c}^{\prime(\prime\prime)}$ in Equation (10). The theoretical dynamic factor limited by the grip is denoted as D_{ag} and illustrated in Figure 3.

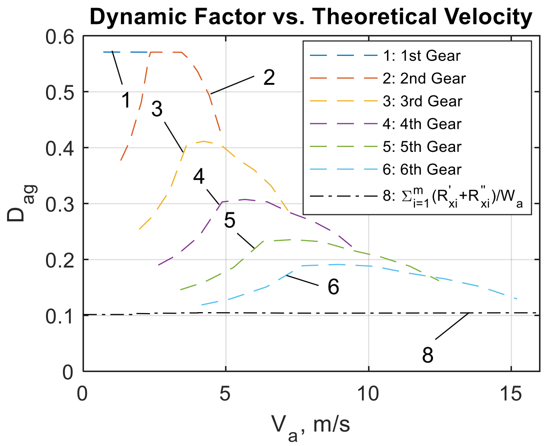

Figure 3: Dynamic factor of FED Alpha, Potential values limited by grip, $\boldsymbol{D_{ag}}$

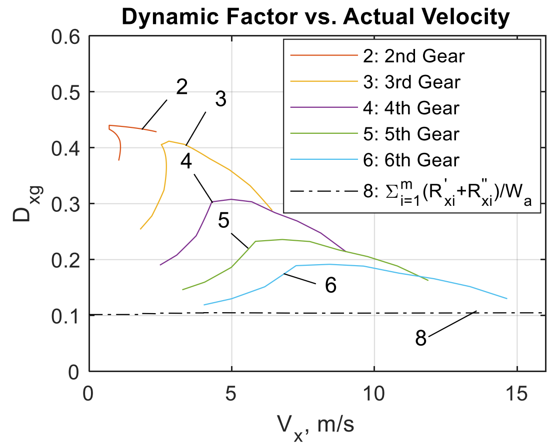

Figure 4: Dynamic factor of FED Alpha, Actual values limited by grip $\boldsymbol{D_{xg}}$

In Figure 3, the vehicle is not able to fully utilize its dynamic potential on the 1st gear in transmission since the tire-terrain grip is not sufficient to take the traction force that can be provided by the powertrain (i.e., the D_{ag}-line 1 in Figure 3 does not reach the maximum it does in Figure 2 and is no higher than the curve for the 2nd gear). The values of the dynamic factor on the 2nd gear are also limited by the terrain properties and cannot be higher than the values given by horizontal line 1 in Figure 3.

As seen in Figure 3, the vehicle can realize its dynamics potential on meadow in full scale beginning from the 3rd gear in transmission. Also, the vehicle has potential to move in steady motion on the highest 6th gear along this terrain. This is because the dynamic factor on the 6th gear is greater than the normalized rolling resistance of $\frac{\sum_{i=1}^{m}(R'_{xi}+R''_{xi})}{W_a}$ represented by curve 8 in Figure 3.

At the same time, the actual values of the dynamic factor and vehicle velocity cannot be seen in Figure 3. The actual dynamic characteristics are given in Figure 4, in which the dynamic factor is a function of the actual velocity of the vehicle as it was computed using the proposed method for determining the vehicle generalized slippage. The actual dynamic factor also limited by the grip is denoted as D_{xg}.

The dynamic factors in Figure 4 are calculated in steady motion with zero acceleration (a_x in Equation (17) is assigned zero). To ensure the balance of forces in Equation (17) results in zero acceleration when the total circumferential force exceeds resistance to motion, a variable drawbar pull force F_d is applied to make the left side of Equation (17) equal zero.

Figure 5 graphically compares the curves from Figure 3 and Figure 4 to illustrate the impact of the tire slippage on the dynamic factor.

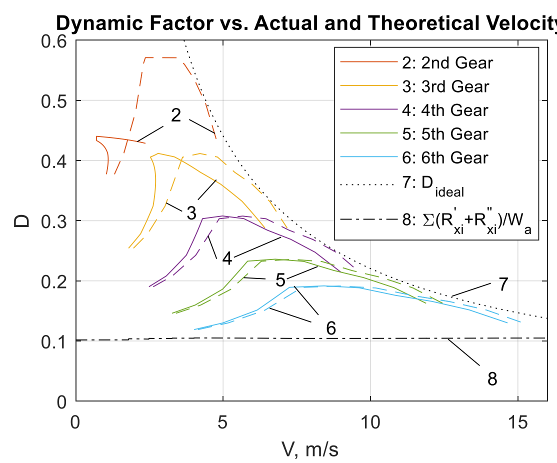

Figure 5: FED Alpha: Theoretical (dash curves) and actual (solid curves) values of dynamic factor

The difference in the values of the dynamic factors, D_{ag} and D_{xg}, increases with lowering the transmission gear. It can be concluded that analysis of the dynamic factor of off-road vehicles that operate mostly at higher values of the circumferential forces and lower gears in transmission should be conducted taking into consideration the driveline characteristics and their impact on the tire slippage values and, thus, vehicle actual velocity.

3. DYNAMIC FACTOR AND ACCELERATION LIMITS

From Figure 5, important *limiting characteristics* can be extracted and analyzed, including the max dynamic factor and its velocity on the highest gear, the max velocity and dynamic factor on the highest gear, and the max dynamic factor on the lowest gear. Table 1 provides those magnitudes. While the grip does not limit the dynamic factor at the highest gear, the drop of the actual dynamic factor on the lowest gear is 49% due to the grip limits and another 22% due to the tire slippages (compare 0.5710 to 1.1287 and 0.4471 to 0.5710).

The theoretical and actual acceleration as another limiting characteristic can be determined from Equation (17):

$$a_a = [D_a - \frac{\sum_{i=1}^{m}(R'_{xi}+R''_{xi})}{W_a}]\frac{g}{\delta_r} \tag{21}$$

$$a_x = [D_x - \frac{\sum_{i=1}^{m}(R'_{xi}+R''_{xi})}{W_a}]\frac{g}{\delta_r} \tag{22}$$

Accelerations in Equations (21) and (22) are computed at zero slope angle θ_n and drawbar pull F_d.

Plots in Figure 6 illustrate these accelerations. As seen from Figure 5, the acceleration and the dynamic factor follow similar patterns.

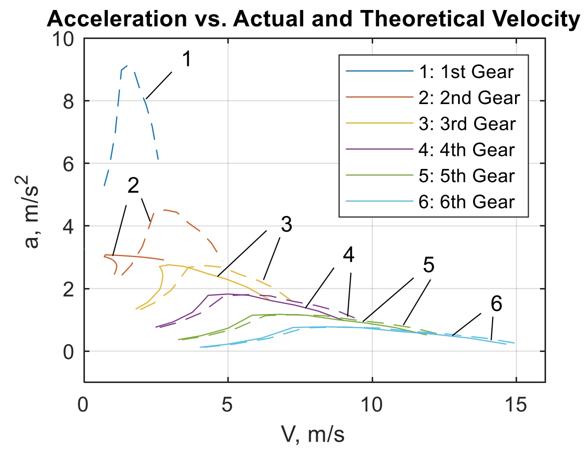

Figure 6: Max acceleration of FED Alpha, Theoretical (dash lines) vs. actual (solid lines) values

Table 2 provides the acceleration limits on all transmission gears.

Table 2: FED Alpha Acceleration Limits

Gear	Max acceleration, m/s²	
	Theoretical, a_a	**Actual, a_x**
1	9.128	-
2	4.510	3.078
3	2.732	2.760
4	1.805	1.824
5	1.164	1.177
6	0.769	0.778

4. CONCLUSION

In this paper, a new approach to analysis of the dynamic factor of off-road multi-wheel vehicles was proposed. The method includes the effects of the distribution of tire slippages, which is dependent on the terrain conditions and driveline characteristics, by computing the vehicle's actual velocity from a vehicle generalized slippage.

Table 1. FED Alpha Maximum Dynamic Factor and Velocity

	Highest Gear				Lowest Gear
Dynamic factor computation	**Max Dynamic Factor**	**Velocity at max dynamic factor, m/s**	**Max velocity, m/s**	**Dynamic factor at max velocity**	**Max dynamic factor**
Theoretical dynamic factor (not grip limited)	0.1914	8.9240	15.1970	0.1297	1.1287
Theoretical dynamic factor (grip limited)	0.1914	8.9240	15.1970	0.1297	0.5710
Actual dynamic factor (grip limited)	0.1916	8.4216	14.6510	0.1301	0.4471

The method can be used to predict military vehicle performance on a terrain of interest through a process of computational simulation. The procedure may be summarized in the following requirements and steps:

1. The condition of the vehicle motion is specified, e.g., steady motion, acceleration, or slope climbing from the cases in Equations (2), (3) and (4).
2. The graph or lookup table of available maximum power vs. speed for a range of gear ratios provides the required characteristics of the vehicle powertrain.
3. Characteristics of the vehicle traction interaction are assigned for the terrain condition as parameters in a traction formula such as Equation (10), which links the individual tire slippages to the tires' vertical and longitudinal forces.
4. A set of equations for the generalized parameters of the driveline configuration is assembled to solve for the vehicle power distribution which determines slippages at the vehicle's tires (example given for the 4x4 FED Alpha in Section 2, where Equations (8) and (9) describe the distribution of circumferential forces in the vehicle driveline and Equation (17) the sum of traction forces and total resistance to motion; F_x-forces in these equations are substituted by their components in Equation (10)).
5. Generalized parameters of the vehicle are computed. The generalized rolling radius of the vehicle in the driven mode r_a^0 is computed with equation (13) from individual wheel rolling radii. The generalized slippage of the vehicle $s_{\delta a}$ is computed with Equation (16) from individual wheel slippages.
6. The generalized slippage and rolling radius are used to determine the speed reduction which predicts the vehicle's actual velocity on the terrain (Equation (14)).
7. The calculations are repeated for the range of the vehicle's engine speed on each transmission gear. The dynamic factors are then plotted against the vehicle actual speed.

By following the above steps, the plot in Figure 4 was obtained. The result shows the maximum dynamic potential at the vehicle's actual speed for each transmission gear. The dynamic factor is a dimensionless value that represents the total potential traction normalized by the vehicle weight. An example was given for the 4x4 FED Alpha.

As shown, including the terrain grip and actual velocity greatly affects the available dynamic factor and acceleration potential at a given velocity when a vehicle is operating at lower gears with high traction. Therefore, slippage and terrain contributions should be taken into account when using the dynamic factor to predict vehicle performance for off-road vehicles in operational conditions requiring large traction.

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6. ACKNOWLEDGEMENTS

This research was funded by Automotive Research Center Phase V - Cooperative Agreement #3.A88, Federal Award W56HZV-19-2-0001, ARC Subaward SUBK00014000